

Harvard Undergraduate Science Olympiad Invitational 2020

# Astronomy C EXAM BOOKLET and ANSWER SHEET

## **Directions**:

- Each sub-question in Q1-7 is worth two points. Q8-10 are worth an ambiguous number of points.
- Partial, integral credit will be given for sub-questions with multiple parts.
- Only the answer sheet will be scored.
- Computational problems will accept a range of numbers.
- Ties will be broken by comparing the points scored on the last sub-questions.
- Questions? Email us at awli@college.harvard.edu or ashernoel@college.harvard.edu!
- Feedback Form Code (for use in HUSO iOS app): HubbleFlowNMT12

Names:

School name:

Team #:\_\_\_\_\_

Score: \_\_\_\_\_

#### Question 1: DSOs 1

1. For the following ten images, identify the:



### Question 2: Light Curves

3. Answer the following sub-questions about the associated image to the left.



#### Question 3: SN UDS10Wil in Depth



- 3. The following questions relate to SN UDS10Wil, which had a B-band absolute magnitude of -19.3:
  - (a) What is the redshift of this supernova?
  - (b) Calculate the scale factor for the universe at the time of the supernova.
  - (c) What drove the expansion of the universe during this time?
  - (d) Use the table to estimate the peak Vega apparent magnitude.
  - (e) The following fit to calculate is used to calculate the corrected apparent magnitude:

$$m_{corr} = m_B^* + \alpha \times (s-1) - \beta \times C$$

where  $\alpha = 1.367, \beta = 3.179$ , and C = -0.071. Use the lower image to estimate the SIFTO stretch parameter.

- (f) Use the fit to calculate the corrected apparent magnitude.
- (g) Calculate the proper distance to the object, in megaparsecs.
- (h) Using your answers in (a) and (d), which model in the top right image is predicted by the supernova?
- (i) What is the predicted energy density of matter by the model that best fits the SN, in MeV/m3?
- (j) Would you expect to find to find H I, S II, and/or H II lines in the DSO's spectrum?
- (k) What is the theoretical maximum mass of the progenitor? Why?

#### Question 4: The Dark Ages



- 4. After decoupling, this universe was dominated by opaque, neutral hydrogen. In cosmological history, this period of time is referred to as the "Dark Ages."
  - (a) What is the qualitative difference between the lyman-alpha forest of quasars at low and high redshifts?
  - (b) What transition produces the Lyman-alpha forest?
  - (c) Dense neutral hydrogen is opaque to the Lyman-alpha forest. Estimate the redshift of reionization, the end of the "Dark Ages."
  - (d) Calculate the size of the universe at re-ionization as a fraction of its current size.
  - (e) Using image E,  $\Omega_m = 0.3$ , and a Hubble constant from Planck of 67.7 km/s/Mpc, estimate the age of the universe at re-ionization.
  - (f) What does (e) suggest about galaxy formation?
  - (g) What does (e) suggest about the existence of hot or dark matter? Why?
  - (h) What does (e) suggest about the validity of the  $\Lambda$ CDM model? Why?

#### Question 5: NGC 1614



- 6. NGC 1614 is a spiral galaxy undergoing an extreme starburst. Hubble observes it to have an apparent magnitude  $m_v$  of 12.91 and a B-V color index of 0.51
  - (a) What is NGC 1614's maximum rotational velocity 275 or 2750 km/s?
  - (b) What is one possible cause of the bump in the extinction curve at 217.5 nm?
  - (c) What is NGC 1614's Hubble type Sa, Sb, or Sc?
  - (d) Calculate NGC 1614's absolute B-band magnitude  $M_B$ .
  - (e) Calculate NGC 1614's absolute visual magnitude  $M_V$ .
  - (f) Use the fact that  $M_{B,sun} = 5.47$  to find the B-band luminosity of NGC 1614 in  $L_{\odot}$ .
  - (g) Use the following two equations to find the extinction at 0.15 microns  $A_{0.15}$ .

$$L_{B,host}(L_{\odot}) = 10^{9.9} \text{SFR}_{starb}^{0.94}$$
$$10^{A_{0.15}} = 250 \times \text{SFR}_{starb}^{2.2}$$

- (h) What is the color of the curve that represents the absolute extinction for NGC 1614 - orange, purple, white, or green?
- (i) Use this curve to calculate the starburst's V-band extinction  $A_V$ .
- (j) Use your previous answers and  $A_V = 1.24$  for the Milky Way to calculate the distance to NGC 1614 in megaparsecs.

#### Question 6: Rapid Recall



#### 7. Identify:

- (a) The nuclear process that creates the black line in the top image.
- (b) The name for the relation shown in the second image from the top.
- (c) The type of black hole related to the third image from the top.
- (d) (6 points) At least one letter (or NA if there are none) in the fourth image from the top where one would expect to find:
  - i. The Sun
  - ii. Horizontal branch stars
  - iii. Main sequence stars fusing hydrogen via the CNO cycle
  - iv. Blue stragglers in a globular cluster
  - v. A black hole
  - vi. The progenitor of a neutron star merger
- (e) (4 points) The term that describes:
  - i. The interaction when two galaxies collide and "eat" each other.
  - ii. The motion of objects solely due to the expansion of the universe.
  - iii. Objects where neutron stars exist completely within a red supergiant.
  - iv. The study of pulsation modes of stars in order to investigate their internal structures, chemical composition, rotation, and magnetic fields.
- (f) (2 points) Consider a theoretical 200  $M_{\odot}$ , Population III, main sequence star. Predict:
  - i. The eventual type of supernova.
  - ii. The eventual remnant.



#### **Question 7: Diagnostic Diagrams**

- 8. Baldwin-Phillips-Terlevich (BPT) diagrams are useful for distinguishing between different ionization mechanisms of nebular gas. One is shown on the top left. It is known that starburst regions produce *weaker* high-excitation lines as compared to other ionization mechanisms (AGNs, LINERs, etc.).
  - (a) What does the bracket notation around NII signify? Briefly explain the cause.
  - (b) (6 points) What is the color of the region on the BPT diagram where one would expect to find:
    - i. The source that created spectrum A (bottom left).
    - ii. The source that created spectrum B (bottom right).
    - iii. A "Green Pea" galaxy.
  - (c) What is the name of the relation shown in the image in the top right?
  - (d) Where would one expect to find source A on the diagram in the top right as a filled circle, open circle, or filled square?
  - (e) Use spectrum B to calculate the redshift z of its source.
  - (f) Calculate source B's recessional velocity, in km/s.
  - (g) A recent paper determines via supernova photometry that the source that created spectrum A is 110.4 Mpc away. Use this to calculate Hubble's constant,  $H_0$ .
  - (h) Use this Hubble constant to calculate the Hubble time of the universe, in Gyr.

**Problem 8 (Tidal Forces).** Suppose we are living in a two-dimensional world where there exists a 1/r gravitational force instead of our  $1/r^2$  force in three dimensions. Let this gravitational force be given by

$$\vec{F}_g = -G_2 m_1 m_2 \frac{\vec{r_1} - \vec{r_2}}{|\vec{r_1} - \vec{r_2}|^2}$$

The tidal force  $\vec{F}_t$  on a 2particle mass m from a 2star mass M is given by the difference between this gravitational force  $\vec{F}_g$  and the fictitious translational force  $\vec{F}_s$ 

$$\vec{F}_t = \vec{F}_g - \vec{F}_s = G_2 M m \frac{\vec{p} - \vec{r}}{|\vec{p} - \vec{r}\,|^2} - G_2 M m \frac{\vec{p}\,^2}{|\vec{p}\,|^2}$$

where  $\vec{p}$  is the vector from the 2particle to the 2star and  $\vec{r}$  is the vector that describes the radius of our 2particle.

**Problem 1.a.** Find the tidal force on a 2particle of mass m on the surface of a circular 2planet with radius r orbiting a 2star with mass M at a distance  $a \gg r$  as a function of the angle  $\theta$  shown below. Take the center of the 2planet to be the origin of your coordinate system and assume that the 2star is at (a, 0). Then  $\theta$  is the angle from the x-axis.

Hint. Manipulate the given equation and Taylor expand around a reasonable variable in calculations.

**Problem 9 (Disintegrating Rods).** Consider the following six-mass system connected by massless rods. The mass' positions are given by:

$$\vec{r}_1 = \left(-\frac{l}{2}, -\frac{l}{2}, \frac{l}{2}\right) \qquad \vec{r}_2 = \left(-\frac{l}{2}, -\frac{l}{2}, -\frac{l}{2}\right) \qquad \vec{r}_3 = \left(\frac{l}{2}, \frac{l}{2}, \frac{l}{2}\right) \vec{r}_4 = \left(\frac{l}{2}, \frac{l}{2}, -\frac{l}{2}\right) \qquad \vec{r}_5 = \left(\frac{l}{2}, -\frac{l}{2}, \frac{l}{2}\right) \qquad \vec{r}_6 = \left(\frac{l}{2}, -\frac{l}{2}, -\frac{l}{2}\right)$$

where l = 1 and each block has mass m with moment of inertia I with respect to all **principal axes**. A principal axis is an axis that arises from symmetry (i.e. any axes perpendicular to a plane of symmetry, any axis parallel to a rotational axis of symmetry). Objects rotating about a principal axis require no additional torques to keep it rotating.



(a) System before disintegration

(b) System after disintegration

Consider the system at (a) rotating with angular velocity

$$\vec{\omega} = \omega_0(\hat{x} + \hat{y})$$

and the system (b) at time t = 0 immediately after some of the rods spontaneously disintegrate.

Problem 2.a. Find the forces and torques of each of the masses immediately after the rods disintegrate.

*Hint.* A small change in momentum  $d\vec{p}$  of any one of the blocks can be described with the **cross product**  $d\theta \times \vec{p}$ 

$$d\vec{p} = d\theta \times \vec{p}.$$

This is because the magnitude of the momentum never changes, only its direction — an infinitesimal change in the momentum is simply an infinitesimal change in the angle of the initial momentum vector. Use this fact, combined with the **no-slip condition** 

$$\vec{v} = \vec{\omega} \times \vec{r}$$

to determine the forces.

**Problem 2.b.** Determine the equations of motion of the "1234-frame" and the "56-dumbbell" as a function of time after disintegration.

Hint. You may want to consider rotation matrices in three-dimensions.

**Problem 10 (Special Relativity).** Special relativity is the generally accepted physical theory regarding the relationship between space and time — spacetime. According to Einstein's original formulation, it is based on two postulates

- The laws of physics are invariant with respect to all inertial reference frames (non-accelerating frames of reference)
- The speed of light c in a vacuum is the same for all observers, regardless of the motion or light source of the observer

Events with coordinates with respect to two reference frames S and S' are related by **Lorentz transforma**tions. These transformations relate the positions, times, and lengths from one reference frame S to another S' moving at a speed v relative to S with the **Lorenz factor**  $\gamma$  given by

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

The Lorentz transformations give us the following relationships/phenomena:

1. Time Dilation: a time difference  $\Delta t'$  measured by a clock in S' is *longer* than a time difference  $\Delta t$  measured by a clock in S.

$$\Delta t' = \gamma \Delta t$$

2. Length Contraction: a length  $\Delta l'$  measured in a moving frame S' is shorter than the length  $\Delta l$  measured in the rest frame S.

$$\Delta l' = \Delta l/\gamma$$

3. Velocity addition: if an object is moving at a velocity  $\vec{u}'$  in the moving frame S', then its velocity  $\vec{u}$  in the rest frame S is given by

$$\vec{u} = \frac{v+u'}{1+vu'/c^2}$$

In relativistic units, we can set c = 1 and add in factors of c at the end of calculations appropriately for units.

**Problem 3.a.** Consider a relativistic rocket length L that is traveling from Earth to Mars at a speed  $v_1$ . A relativistic astronaut walks from the rear to the front of the rocket with a speed  $v_2$  with respect to the rocket.

How long does the astronaut's walk take according to a clock at the rear of the ship?

Problem 3.b. How long does the walk take according to the astronaut's clock?

#### Answer Sheet A: Questions 1-4





#### Answer Sheet B: Questions 5-7



## Answer Sheet C: Questions 8-10

8. (a)

9. (a)

(b)

10. (a)

(b)